

## A Simplified Muon Lifetime Experiment for the Instructional Laboratory\*

R. E. HALL, D. A. LIND, AND R. A. RISTINEN

*Department of Physics and Astrophysics, University of Colorado, Boulder, Colorado 80302*

(Received 27 April 1970)

An experiment has been developed for the measurement of the half-life of cosmic ray muons by using a single scintillator detector. The associated electronics are much simplified over conventional cosmic ray muon lifetime measuring systems. It is this simplification that makes the experiment well suited for the instructional laboratory.

### I. INTRODUCTION

Lifetime measurements of cosmic ray muons are commonly performed with some type of multiple detector telescope.<sup>1</sup> A typical arrangement is shown in Fig. 1. With this system counters 1, 2, and 3 have their output signals routed such that the time-to-amplitude converter (TAC) is started only by muons that pass through 1 and 2 but not 3. Because of the geometry of this arrangement, a start signal to the TAC indicates that a charged particle has stopped in the absorber. If the stopped particle was a muon, the subsequent decay into an electron plus a neutrino pair sends a stop pulse to the TAC, and an output pulse representing the start-stop time is recorded by a pulse height analyzer (PHA). Since the half-life of a free muon is  $1.53 \mu\text{sec}$ ,<sup>1</sup> corresponding to a mean life of  $2.21 \mu\text{sec}$ , the chance of a decay occurring per interval of time decreases by half after each  $1.53 \mu\text{sec}$  of time. Therefore, the TAC pulses stored in successive channels of the PHA will represent an exponential decay curve. By plotting the successive channel accumulations on semilogarithmic graph paper, a straight line of negative slope is produced. The straight line nature of the graph indicates that a radioactive decay phenomenon is indeed being observed, and by fitting a straight line to the semilog graph of the data points, one may easily extract the half-life of the muon.

While this experimental setup for measuring the cosmic ray muon half-life is logical in arrangement and has worked well for a number of experimenters with various modifications, it is relatively complex and expensive. In this paper we discuss the development of a simplified apparatus which yields essentially the same lifetime measurements as the multiple detector arrangement just described.

### II. APPARATUS AND ITS OPERATION

The simplified system is shown in Fig. 2. Its operation is initiated when a muon traverses a portion of the plastic scintillator and comes to rest. The energy lost in the stopping process ionizes the plastic and gives rise to a pulse of light which is transformed into an electrical pulse and amplified by the photomultiplier tube. The signal for the discriminator is brought directly off the anode of the photomultiplier tube by a  $50\text{-}\Omega$  cable and is proportional in amplitude to the intensity of the light pulse. As a result, if the incident muon traverses enough of the scintillator plastic, the electrical pulse delivered to the discriminator will cause it to trigger. The triggered discriminator sends out a  $4\text{-nsec}$  wide pulse which is then equally divided between two  $50\text{-}\Omega$  cables which drive the start and stop inputs of the TAC. The cable to the start input of the TAC is approximately 8 ft longer than the cable to the stop input. In its normal state, the TAC is ready to accept only a start pulse. It ignores the stop pulse, and the start pulse starts a timing run. The delay of the start pulse relative to the stop pulse by the 8 ft of additional cable assures that the stop input is no longer activated when the pulse gets to the start input. Failure to include this delay time would mean that the TAC would be unable to accept the start pulse. It should be noted that the 8-ft cable extension subtracts approximately 8 nsec from the time to which each PHA channel would normally correspond. This does not change the shape of the spectrum stored in the PHA, and the half-life obtained from the slope of the spectrum will be unchanged.

A muon that started the TAC timing run may be stopped in the scintillator and decay at some later time. The neutrinos from the decay leave the scintillator without any further effect. The

electron leaves an ionization trail in the scintillator, and as a consequence, the decay of the muon causes another input signal to go to the discriminator. This pulse at the stop input of the TAC causes it to measure the time since the last start pulse and to produce a bipolar output pulse of 2  $\mu\text{sec}$  width with an amplitude linearly proportional to this time. The amplitude information is in turn recorded by the PHA. The read out process takes much longer than the propagation of the decay electron signal from the stop to the start inputs, and as a result, the corresponding start pulse will be ignored.

The only piece of equipment needed to perform the experiment which is not available as a standard unit is the scintillator-photomultiplier tube assembly. Because the natural cosmic ray flux is not very intense, approximately  $1 \times 10^{-2}$  muons/cm<sup>2</sup> sr sec at sea level, and about twice this value at the 5000-ft elevation of the University of Colorado, the need for a scintillator with a large horizontal area is indicated. Also, since the energy spectrum of the muon flux reaches a maximum in the neighborhood of 1 BeV, the scintillator should be as thick as possible. These considerations and the availability of a large piece of scintillator plastic resulted in a  $38 \times 24 \times 23$  cm rectangular block being cut for the apparatus. An RCA 7264 photomultiplier tube was optically coupled to one of the small ends of the scintillator block. The entire assembly was then wrapped with aluminum foil and black

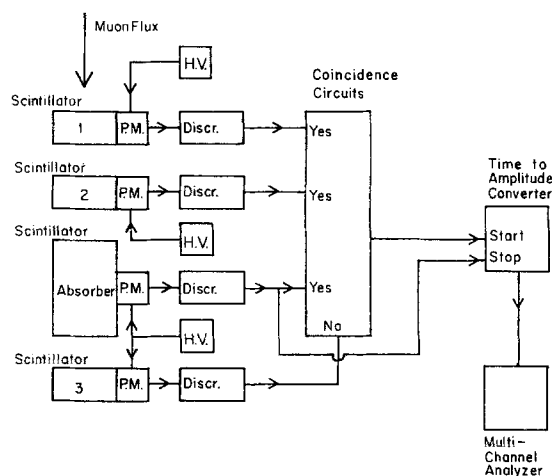


FIG. 1. A typical muon lifetime experimental setup. Details of its operation are discussed in the text.

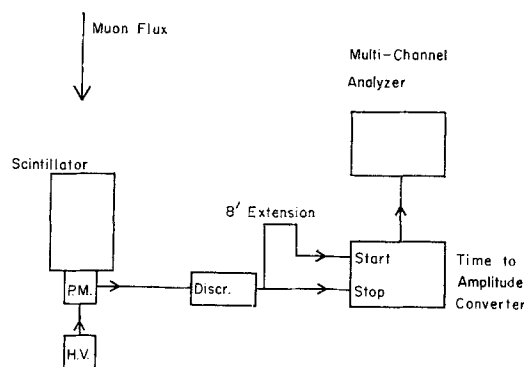


FIG. 2. Simplified setup used for this experiment. The particular items of equipment used are identified as follows: photomultiplier tube: RCA 7264; discriminator: Ortec model 417; TAC: Ortec model 437; Multichannel analyzer: Nuclear Data Corp. model ND130A.

tape. Finally, an aluminum frame was constructed to support and protect this assembly and hold the photomultiplier tube base.

After the components of the system have been connected, the PM tube high voltage, the discriminator level, and the TAC time range must be adjusted. The high voltage was selected by setting the discriminator to its lowest threshold, about  $-0.18$  V and then increasing the voltage applied to the PM tube until a reasonable trigger rate was obtained for the discriminator. At  $-2010$  V, a trigger rate of 150/sec was obtained. To obtain a clear decay curve, the discriminator setting must be raised. A discriminator setting of  $-0.8$  V was found to give the highest possible TAC output rate and yet provide data representing a smooth decay curve.

### III. RESULTS

A long experimental run was made for a period of 695 h with a TAC range of 20  $\mu\text{sec}$  and a discriminator setting of  $-0.8$  V to see where the decay curve merges into the background which is due to discriminator pulses randomly occurring within 20  $\mu\text{sec}$  of each other. This same run was used as a best measurement of the half-life of the muon. For the 695-h experimental run, a pulser capable of producing pulse pairs was used along with an oscilloscope to establish a time to PHA channel gain of  $0.112 \pm 0.003$   $\mu\text{sec}/\text{ch}$ . After 695 h, the spectrum of Fig. 3 resulted. Both the muon decay curve and the background are clearly evident. Channels 160–180 were averaged

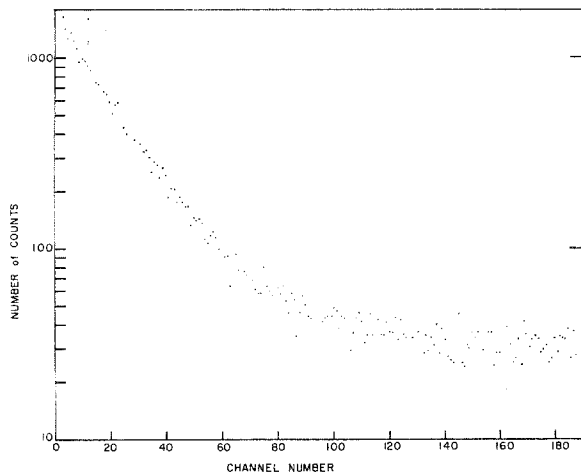


FIG. 3. Data from a 695-h run before subtracting background. The time width of each channel is  $0.112 \pm 0.003 \mu\text{sec}$ .

to obtain a background of  $31 \pm 2$  counts. Subtraction of this background gives the time spectrum of Fig. 4. The use of a computer to do a least squares fit of a straight line to these data gives a half-life of  $1.46 \pm 0.04 \mu\text{sec}$ . (It should be noted that the accepted half-life of the free muon is  $1.533 \pm 0.001 \mu\text{sec}$ .) The measured value may in fact be under the accepted value because about 45% of the muon flux is  $\mu^-$  and 55%  $\mu^+$ .<sup>2</sup> The  $\mu^-$  particles have a reduced half-life relative to  $\mu^+$  particles in matter since the  $\mu^-$  particles are captured into atomic orbitals which allow interactions with the nuclei of the matter. Thus, the observed half-life might be expected to be somewhat less than that of free muons, with the exact reduction in lifetime determined by the unknown probability with which a nuclear capture of a muon results in a discriminator pulse. In this work, we have chosen to approximate what may in fact be a complex decay curve by a straight line exponential decay.

The background that was subtracted from the 695-h spectrum results from the rather high discriminator trigger rate of 10.4/sec. Since the analyzing rate was only 0.01/sec over-all, almost all of these discriminator pulses were random. Theoretically, the background per channel of the 695-h spectrum should be:

$$\text{Background} = (\text{discriminator rate})^2 (\text{time width of channel}) (\text{analyzing time}).$$

Evaluation gives

$$(10.4)^2 (1.12 \times 10^{-7} \text{ sec}) (2.5 \times 10^6 \text{ sec/sec}^2 \text{ ch.})$$

$$= 30.2 \text{ background counts/ch.}$$

This agrees with the observed background of  $31 \pm 2/\text{ch}$ .

Since a 695-h run is almost 29 days long, a much shorter run would be necessitated by the schedules of most instructional laboratory courses. A 24-h run was made with a PHA channel width of  $0.0329 + 0.0009 \mu\text{sec}/\text{ch}$ . to determine the quality of the data to be obtained in this time. Figure 5 represents the results of this run. The background is less than one count/channel and has been ignored. The short time span of the spectrum also reduces the significance of any background. A least squares fit of a straight line to these data gives a half-life of  $1.64 \pm 0.15 \mu\text{sec}$ . For a run of this short duration a more accurate time base determination is not needed as the largest error associated with the half-life is due to statistical variations in the data.

It is clear that aside from improving the time base calibration, the greatest improvement in the performance of the experimental setup would occur if the analyzing rate to discriminator trigger rate ratio could be improved. For the 695-h run,

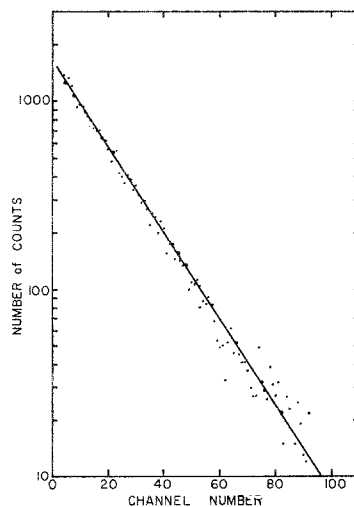


FIG. 4. Data from a 695-h run after subtracting 31 counts of background from each channel. The time width of each channel is  $0.112 \pm 0.003 \mu\text{sec}$ . A computer fit of a straight line to the data points has been drawn in. Its slope represents a half-life of  $1.46 \mu\text{sec}$ .

it was approximately  $1 \times 10^{-3}$ . It is low for the following reasons. First of all, the muon flux peaks in intensity somewhat above 1 GeV. This means that a large number of muons can easily pass through the scintillator which can only slow a muon down by approximately  $2 \text{ MeV/cm} \times 38 \text{ cm} = 76 \text{ MeV}$ . In order to trigger the discriminator with a  $-0.8 \text{ V}$  setting, approximately 10 MeV of energy must be deposited in the scintillator. Consequently, almost all muons passing through the counter contribute to the trigger rate without contributing to the analyzing rate. Also, those muons which stop in the scintillator may decay without providing a stop pulse to the TAC if less than 10 MeV of energy is deposited in the scintillator in the decay process. The momentum of electrons or positrons produced by muon decay has a continuous distribution and is described by Fig. 6.<sup>2</sup> Since the neutrinos do not interact with the scintillator, we need only be concerned with the electrons. A small number of the decay electrons produced in the scintillator will not have sufficient energy to trigger the discriminator. Also, some of those with more than 10 MeV of energy may escape the scintillator before they deposit 10 MeV of energy. The deposition of 10 MeV requires approximately 5 cm of

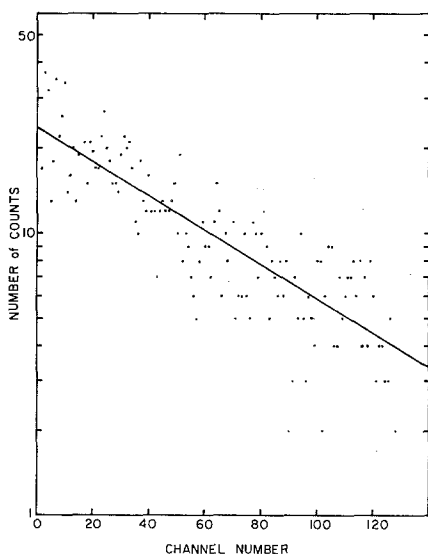


Fig. 5. Data from a 24-h run. The time width of each channel is  $0.0329 \pm 0.0009 \mu\text{sec}$ . A computer fit of a straight line to the data points has been drawn in. Its slope represents a half-life of  $1.64 \mu\text{sec}$ .

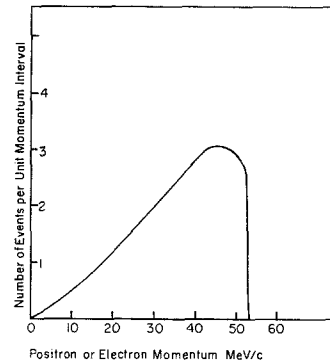


Fig. 6. Momentum distribution of electrons or positrons produced by muon decay. Taken from Ref. 2.

plastic, which means that electrons of more than 10 MeV of energy can fail to trigger the discriminator if they are formed within 5 cm of the side of the scintillator. Another very small contribution to the trigger rate and not the analysis rate arises from those muons which enter the scintillator with less than 10 MeV of energy. Only a decay pulse can then be observed, and these particles therefore contribute to the random rate.

This information would indicate that the average muon encountering the scintillator will produce one discriminator pulse and not two. In general, one would expect that the ratio of analyzing rate to discriminator trigger rate would be increased if the scintillator were made larger.

It would be interesting to examine the correspondence between the rate at which muons encounter the scintillator and the discriminator trigger rate. The muon flux will be taken as  $2 \times 10^{-2} \text{ muons/cm}^2 \text{ sr sec}$ . Because the scintillator is oriented with its longest axis vertical and the muon flux is most intense near the vertical, the usable solid angle will be taken as 2 sr. The available detector area is  $23 \times 24 \text{ cm}^2 = 552 \text{ cm}^2$ . The discriminator trigger rate should be of the order of

$$\frac{2 \times 10^{-2} (552 \text{ cm}^2) (2 \text{ sr})}{\text{cm}^2 \text{ sr sec}} = 22.1/\text{sec}.$$

To obtain discriminator trigger rates in excess of this value, the discriminator level must be lowered to about  $-0.4 \text{ V}$ . It is also at discriminator levels below  $-0.4$  to  $-0.5 \text{ V}$  that the time

spectrum becomes badly nonlinear. This would indicate that the discriminator is probably triggering due to meaningless noise from the photomultiplier tube at these low settings.

#### IV. SUMMARY

This experiment performs well and is simple enough to be used in an undergraduate physics lab. With some care given to least squares methods of data analysis and accurate time calibration, it is capable of producing research quality data.

#### ACKNOWLEDGMENTS

The assistance of Mr. Harry Clark in machining the scintillator plastic and the help of several other staff members in the development of this apparatus is sincerely appreciated.

\* Equipment provided in part by a National Science Foundation Instructional Scientific Equipment Grant.

<sup>1</sup> A. C. Melissinos, *Experiments in Modern Physics* (Academic, New York, 1967).

<sup>2</sup> B. Rossi, *High Energy Particles* (Prentice-Hall, Englewood Cliffs, N. J., 1952).

### Visual Appearance of a Moving Vertical Line

RAMESH BHANDARI

*Physics Department, Panjab University, Chandigarh, India*

(Received 2 March 1970; revision received 19 May 1970)

A vertical line moving with velocity  $v$ , when seen, assumes the shape of a hyperbola or parabola accordingly as  $v < c$  or  $v = c$ . At  $v > c$ , which actually is not possible, the line takes the form of an ellipse, or parts of an ellipse, or even becomes imaginary, depending upon the length of the line, the magnitude of the velocity, and the distance of the viewer from the line. If, however, a line moving with  $v < c$  recedes from the viewer, it seems less curved until at infinity it straightens up to look vertical again.

In this paper, I have dealt in detail with the appearance of a moving vertical line and the subsequent changes that take place in it with the passage of time.

Consider a line coincident with the  $z$  axis of a frame moving with velocity  $v$  as shown in Fig. 1. The light ray I from end 0 reaches the viewer's eye fixed in the laboratory frame at  $(0, y, 0)$  after a time interval  $y/c$ . The ray II from  $A$ , on the other hand arrives at  $P$  at the same time as I after being emitted at an earlier instant  $\Delta t$ . From the figure, it is clear that

$$c\Delta t + y = [(v\Delta t)^2 + y^2 + Z^2]^{1/2}. \quad (1)$$

Multiplying both sides by  $v/c$  and squaring the resulting expression, we arrive at the equation

$$X^2(1 - \beta^2) + 2X\beta y - \beta^2 Z^2 = 0, \quad (2)$$

where  $X = v\Delta t$ . This can be further put in the form

$$[(X + \beta\gamma^2 y)^2 / (\beta\gamma^2 y)^2] - [Z^2 / (\gamma y)^2] = 1. \quad (3)$$

Equation (3) is in fact the equation of the locus

of all points lying on the line, the light rays from which, emitted at different instants ( $\Delta t$ ), arrive at  $P$  simultaneously. It is similar to the standard equation<sup>1</sup> of a hyperbola in the  $xz$  plane, drawn with the origin shifted to  $(x^1, y^1)$ . To the viewer, therefore, the line looks like a hyperbola with focus  $F \equiv [y/(1 + \beta), 0]$  and directrix  $D$  given by

$$x = -\beta y / (1 + \beta); \quad (4)$$

$y$  here is a constant. The eccentricity  $e$  is a function of  $v$  and is equal to  $1/\beta$ .

#### DEPENDENCE OF APPEARANCE ON $v$

At  $v = 0$ ,  $e = \infty$ .  $F$  tends to  $(y, 0)$  and the directrix coincides with the  $z$  axis. The hyperbola, consequently, straightens up coinciding with the  $z$  axis. However, when the line moves with a finite velocity  $v < c$ , it assumes the shape of a hyperbola and at  $v = c$  transforms into a parabola given by

$$Z^2 = 2yX. \quad (5)$$

It is interesting to note that with further increase